Event-Driven Finance

IEOR – Fall 2017

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Suppose you are working for a hedge fund and someone comes to you with a curious observation. They wish to know if their observation is
- anomalous?
- regular?
- tradeable?
You are armed with a database and your intuition
How should you proceed?
An Apple Conspiracy? Theories on That $500 Close

By Nick Summara on January 22, 2013

Shares of Apple (AAPL) closed at 500 on Friday, Jan. 18. Not 499.99, not 500.01—five zero zero point zero zero zero dollars on the nose. There's a long history of market watchers having cried conspiracy on Apple stock and for some observers, the impossibly round number was just too much of a coincidence. "I still have that bridge to sell you if you don't think the fix was in on this," wrote John Gruber an Apple über-blogger.

A Twitter chorus joined in:
• Proof of stock market manipulation
• If this doesn't merit an SEC investigation then they should just close
• Can't imagine all the crazy back-house trading and manipulation that must have occurred to have $AAPL land exactly at $500,000
• I’m reminded again why amateurs shouldn’t get involved in the financial markets

For some, the neat 500 close seemed all the more fishy for coming so soon after loosely sourced reports of weak
Pinning

KO pinning to 67.50 (weeklies)
Lecture 3f

Pinning

KO: Oct 15, 16, 17

Price

Tick
Today we want to look at a static property of the option markets.

Not all phenomena which appear to violate “standard” option theory are dynamic. As you know, there are many assumptions made in standard classical finance which we know, or suspect, cannot hold in the real markets.

Suppose you see the following market:

<table>
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<th>XYZ</th>
<th>Jun 40 C</th>
<th>8.50 – 8.80</th>
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<td>(Underlying)</td>
<td>48.46 – 48.52</td>
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Expiration day.

First of all, what does this mean? What is the fair value of the calls?

Classical theory says that the Jun 40 calls are overpriced. By how much? Why haven’t they traded?
• Costs are an obvious area typically ignored in order to price options.
• A more subtle idea is the assumption of a stock process. This is a stochastic process for the stock, independent of the presence of options trading.
• Suppose someone bids for 25000 calls all at once. (On Friday, April 28, 2006 this happened in MSFT May 25 (at-the-$) calls.) Do you suspect that the stock would move in a correlated fashion? Which way? (In MSFT the stock price moved from 24.05 to 24.17 in 15 minutes from the origin of the order.)
• This means that on certain time scales a demand for (supply of) stock moves the stock. Quantifying this effect theoretically means identifying an Impact Function.
• What about the very presence of outstanding option open interest? Typically it would seem not, because undoubtedly positions are hedged. And yet, sometimes option positions lead to changing deltas.
• Suppose you hold an XYZ Jun 40 C; it is expiration day and the stock is at 40.35 at 10:30. You calculate the delta and find it is 58.
• At 1:30, three hours later, the stock is still at 40.35. What has happened to the delta of the call? When you recalculate the option delta, it is now 66. Why?
• To stay delta-neutral you must sell an additional 8 shares.
• Now couple this to the assumption that supply (demand) of the stock pushes the stock down (up) and the changing deltas of the option lead to long option holders selling the stock.
• An analogous argument applies with the stock below the strike; now buyers push the stock up toward the strike.
• In the Black-Scholes, classical world, there are an equal number of short option holders doing the exact opposite thing. The net effect should be zero.
• But is this an accurate assumption? Market makers are generally active hedgers. When they are long a strike they aggressively hedge, especially close to expiration. But when they are short a strike and since they cannot continuously hedge, they avoid hedging as long as possible.

• Consider the region over which the delta is changing most rapidly. This is also the region where $\theta \equiv -(\partial C/\partial t)$ is largest. So there is an incentive for a trader to avoid hedging his short option, as long as the possibility of pinning remains high. On the other hand, the long option holder risks losing all the option value to pinning.

• So unlike the Black-Scholes world, real hedging strategies are asymmetric. Coupled with an additional non-classical assumption of stock price movement to supply/demand, there is the possibility of pinning the stock at expiry, that is a non-zero probability of the stock exactly closing at a strike price.
What is stock pinning?

• At the expiration of options, the close of trading on the third Friday of each month, a stock is pinned if it closes exactly at a strike price.

• For practical reasons, pinning can be considered to have occurred if the closing price is close to a strike (±$0.25, say)

• Mathematically: \( P\{|K-S|<\varepsilon\} > 0 \) at expiration for all \( \varepsilon>0 \).
Pinning on Option Expiration Dates

Stock B pinned
Stock A did not
Talking with options traders yields plenty of anecdotal evidence for pinning:

“I can tell when a stock will pin; it’s when I’m long the strike!”

*Michael Acampora* (Gargoyle Strategic Investments)

Although this is not the best of evidence, the apparent high frequency of stock pinning led three independent groups to examine the phenomenon over the last several years.
• Krishnan and Nelkin, RISK December 2001
• Ni, Poteshman and Pearson, UI (August 2004 *electronic form*)
• Avellaneda and Lipkin, Quant. Finance 3 (2003), 417-425
• (many groups subsequent to our work)

• These three groups had strikingly different approaches to the problem which we will discuss later.

• Importantly, the Illinois work, done independently of ours, emphasized real stock data using the IVY database and analyzed trader data via the CBOE data-base. We will reproduce some of this work in the problem set...as well as additional work examining specifics of the A-L model.

• The KN paper, while principally suggesting a model, demonstrated pinning in MSFT.

Intraday volatility declines

JDEC in March 2001

Large sale of options on this day
Large trade *initiates* regime

**JDEC 2001 Mar 10**
**Put & Call Open Interest**

*Average traded vol in stocks = 1MM shares*

*Notional number of shares corresponding to OI = 5.6 MM shares*
UI Urbana Study: Optionable vs. Non-Optionable Stocks

- At least 80 expiration dates
- 4,395 optionable stocks on at least one date
- 184,449 optionable stock-expiration pairs
- 12,001 non-optionable stocks on at least one date
- 417,007 non-optionable stock-expiration pairs
Several results from the UI group. Data from January 1996 through September 2002

Percentage of optionable stocks closing within $0.25 of a strike price

Relative Trading Date from Option Expiration Date

(Courtesy: Ni, Pearson & Poteshman)
Percentage of *optional* stocks closing within $0.25 of an integer multiple of $5

Relative Trading Date from Option Expiration Date

(Courtesy: Ni, Pearson & Potoshman)
Percentage of optionable stocks closing within $0.125 of a strike price

Relative Trading Date from Option Expiration Date

(Courtesy: Ni, Pearson & Poteshman)
Percentage of non-optionable stocks closing within $0.125 of an integer multiple of $5

Relative Trading Date from Option Expiration Date

(Courtesy: Ni, Pearson & Poteshman)
Percentage of non-optionable stocks closing within $0.25 of an integer multiple of $5

Relative Trading Date from Option Expiration Date

(Courtesy: Ni, Pearson & Poshenshman)
Non-optionable stocks that were previously optionable closing within $0.125$ of an integer multiple of $2.50$
• So there is plenty of evidence for pinning, but only in *optionable* stocks. **What models might suffice to explain the effect?**

• Krishnan and Nelkin attack the problem of pinning by assuming that there exists an a priori mixture of pinning paths and independent random walks for the stock price. This model can get any desired probability of pinning, but leaves unanswered how actual option data and parameters, and stock price, may affect the probabilities. Also, once the KN mixture is fixed, the price of the straddle cannot be accurate for all eventual stock paths.

• Ni, Poteshman, Pearson originally suspected **collusion** on the part of market participants. (Post our work, somewhat less so.)
• Which of the following three slides doesn’t belong?
• (And what are they?!)
Lecture 3f

Pinning
Pinning
Lecture 3f

Pinning

THE OPERA HOUSE MASSACRE

Morphy vs Duke of Brunswick and Count Isouard/ Paris Opera House 1858

Paul Morphy was the first great American Chess player. It has been said that no one has been as good at open games as Paul Morphy. He was a tactical and strategic genius decades ahead of his time. Morphy never considered himself a professional player and this game, like a great majority of Morphy’s games, were casual non-tournament games. This Opera Game is probably the most famous game in chess history.

See Morphy Biography

This is on our list of most famous games
• The answer is: the Eiffel tower. Both the termite mounds and the chess game are constructs of independent agents. In other words, although both those slides show a very specific final ordered result, they are the consequence of two or many agents playing out a game. NO MASTER ARCHITECT exists.

• In the game of options trading, individual market-makers play at HEDGING their positions. They do not collude to maintain unbalanced positions.
• All possible models cannot be known, but one which involves market-makers acting independently to maintain approximately delta-neutral positions satisfies Occam’s razor. It requires the fewest assumptions about the outside world. A kind of greatest entropy model.

• It should be noted that there are two distinctions which may be drawn between market participants. Some, market-makers and desk proprietary traders among them, are active hedgers. Others, investors and positional traders, put on positions (often but not always long delta), and let them play out.

• This asymmetry will be important.
• A number of groups have examined the response of markets to orders entering an order book.

• One group is associated with J D Farmer:


• Another group is associated with JP Bouchaud (CFM).
These groups all agree on the common sense notion that BUYING stock raises the market price, and SELLING stock lowers the market price.

Curiously they all disagree on the functional way in which the changing market varies with S/D. (This will be a subject for discussion later.)

\[ \Delta S/S = f(Q) = EQ + E_2 Q^2 + E_3 Q^3 + \ldots = EQ + g(Q), \]
g analytic. This is a simple Taylor’s expansion for market price change as a function of the demand for (supply of) stock. For simplicity, we throw out g(Q) and simply assume a linear form.
• An Interlude for mathematical finance modelers…
• Summarizing the JDEC (and KO) case(s), we have:
  – a very large open interest on the strike which the stock eventually pinned to
  – intra-day volatility seems to have declined markedly after the open interest increased greatly
Following the outline of last time’s lecture, let’s divide the phase space into three regions:

a) $0 < t < t_1$  \textit{no open interest}

b) $t_1 < t < T$  \textit{large, fixed open interest};

\hspace{1cm} $t_1 = \text{large trade initiated}, T = \text{expiry}$

c) $t > T$  \textit{options have expired}

We assume that pricing is of a standard form in a) and c)

We look to model pricing in region b)
Slow variables

- For this approach to be successful, we need to identify a slow variable

- *Slow* means that pricing can be done in a standard fashion imagining all variables are essentially constant, although $S$ will technically be a functional rather than a function and one of the variables will in fact be a function of $t$
• In the case of pinning we will want to treat delta, \( \delta \), as the slow variable
• We have two basic ansatzes:

1. A class of *independent*, long option holders comprises active hedgers
2. Trading stock to hedge options impacts stock price via an impact function
• What we construct in this fashion is essentially a feedback mechanism of independent agents

• Trader ↔ stock ↔ stock price ↔ Trader

• But for the purposes of this approach it is only necessary to imagine 1 agent hedging the entire outstanding delta position
As time advances, the delta of an option (not exactly at the money) moves away from 50 and toward 0 or 100.

Hedging requires a repeated selling or buying of stock which positively impacts the stock price and drives it toward the strike.

We follow the math now...
Estimating the Demand for Deltas using Black-Scholes

\[ \Delta \delta = \frac{\partial \delta}{\partial t} \, dt, \quad \tau = T - t \]

\[ \delta = 2N(d_1), \quad d_1 = \frac{1}{\sigma \sqrt{\tau}} \left( \ln \left( \frac{S}{K} \right) + \left( \mu + \frac{\sigma^2}{2} \right) \frac{\sqrt{\tau}}{2\sigma} \right) \]

From Black-Scholes

\[ y = \ln \left( \frac{S}{K} \right), \quad a = \mu + \frac{\sigma^2}{2} \]

\[ \frac{\partial \delta}{\partial t} = -\frac{1}{\sqrt{2\pi}} \frac{y - a \tau}{\sigma^3 \tau^{3/2}} e^{-\frac{(y+a\tau)^2}{2\sigma^2\tau}} \]
Taking into account demand for stock: Price-Impact Functions

\[
\frac{dS}{S} \propto E\left(\frac{D}{\langle V \rangle}\right)^p \quad \frac{D}{\langle V \rangle} \gg 1
\]

- $p = 0.22$  
  Farmer, Lillo, Mantegna

- $p = 0.5$  
  X. Gabaix

- $p = 1$  
  linear model, (A. & Lipkin)

- $p = 1.5$  
  convex model (Bouchaud, …)
Dimensionless Model for Power-Law Price-Impact Function ($p > 0$)

Price change = Price impact + noise

\[
\frac{dS}{S} \propto -\text{const.} \left| \frac{\partial \delta}{\partial t} \right|^p \text{sign} \left( \frac{\partial \delta}{\partial t} \right) dt + \sigma dW
\]
Impact functions

- The power, $p$, in the previous slides is included to suggest the possibility of a spectrum of (non-analytic) impact functions.
- Work by R. Cont supports the value 1.0 for $p$.
- $p$ may be thought of as a measure of the competition between diffusion and pinning pressure. As $p$ decreases, the impact of hedging becomes less and less.
- Viewing this as a physicist would, we should typically expect a phase transition in the $p$-parameter space from pinning to non-pinning as $p$ declines.
- If this is the case (we shall see it is), then the experimental fact of pinning should constrain the possible impact models.
You may have noted the use of BS for the calculation of delta in the demand equation.

This returns us to our initial discussion:

- We look for simple modular approaches to pricing where the hard part has been moved to the boundaries.
- Too often the presence of market events is used to justify a complex stochastic model designed to price an entire state space.
- The crux of the approach I am outlining here is to use the simplest (Occam) sufficient model with the most comprehensive boundary conditions - the boundaries being selected by the events themselves.
The linear model

Dynamics for Stock Price

\[ \frac{dS}{S} = \frac{E.OI}{\langle |D| \rangle} \frac{\partial \delta}{\partial t} \, dt + \sigma dW \]

\[ y = \ln \left( \frac{S}{K} \right) \]

\[ dy = -\frac{E.OI}{\langle |D| \rangle \sqrt{2\pi}} \cdot \frac{y - a(T-t)}{\sigma(T-t)^{3/2}} e^{\frac{(y+a(T-t))^2}{2\sigma^2(T-t)}} \, dt + \sigma dW \]

- `coupling constant`
- restoring force
- bounded support
- noise
The linear model

Dimensionless Variables

\[ z = \frac{y}{\sigma \sqrt{T}}, \quad s = \frac{t}{T}, \quad z_0 = \frac{y_0}{\sigma \sqrt{T}} = \frac{1}{\sigma \sqrt{T}} \ln \left( \frac{S_0}{K} \right) \]

\[ \alpha = \frac{\sigma \sqrt{T}}{\sigma}, \quad \beta = \frac{E.OI}{\langle D \rangle \sqrt{2\pi\sigma^2 T}} \]

\[ dz = -\beta \frac{z - \alpha(1 - s)}{(1 - s)^{3/2}} e^{-\frac{(z + \alpha(1 - s))^2}{2(1 - s)}} \, ds + dW \]
Dimensionless variables

- $z$ represents the dimensionless (logarithmic) distance to the strike; it’s presence in the formulation insures that the likelihood of pinning is subject to a feedback of the stock price itself.

- $\beta$ describes the strength of the pinning force. It is proportional to the open interest, OI, and the unknown elasticity constant, $E$, and inversely proportional to the stock volatility, $\sigma$.

- $\beta$ represents the strength of the coupling to the “pinning field”
  - You can think of OI as charge, $E$ as the dimensionful coupling constant, and $\sigma \sqrt{T}$ as a temperature.

- $\alpha$ the drift term we will arbitrarily set to 0.
Dimensionless Model (alpha=0) for Linear Price-Impact Function

\[ dZ = -\frac{\beta \cdot z}{(1-s)^{3/2}} e^{\frac{z^2}{2(1-s)}} ds + d\overline{W} \]

Linear restoring force with increasing coupling with time and compact support.
Cumulative PDF for price at expiration date (Beta=0.1)

Pinning naturally appears in this model
Solving the linear response model (p=1)

Assume Alpha=0

Forward Fokker-Planck equation:

$$\frac{\partial F}{\partial s} + \frac{1}{2} \frac{\partial^2 F}{\partial z^2} - \frac{\beta z}{\tau^{3/2}} e\frac{z^2}{2\tau} \frac{\partial F}{\partial z} = 0, \quad \tau = 1 - s$$

Look for solution of the form:

$$F(z, s) = \exp \left( \frac{1}{\sqrt{\tau}} \phi \left( \frac{z}{\sqrt{\tau}} \right) \right), \quad \phi(\varsigma) \text{ unknown, } \quad \varsigma = \frac{z}{\sqrt{\tau}}$$
ODE for the ‘Phase Function’ (WKB)

\[ \frac{\phi + \xi \phi' + \phi''}{2 \tau^{3/2}} + \frac{(\phi')^2 - 2 \beta \xi \phi' e^{\frac{\xi^2}{2}}}{2 \tau^2} = 0 \]

\[ O(\tau^{-2}) \]
\[ (\phi')^2 - 2 \beta \xi \phi' e^{\frac{\xi^2}{2}} = 0 \] \quad \text{Eikonal Equation}

\[ \therefore \quad \phi(\xi) = -2 \beta e^{\frac{\xi^2}{2}} + c \]

\[ O(\tau^{-3/2}) \]
\[ \phi + \xi \phi' + \phi'' = c \quad c = 0 \]

\[ F(z, s) = \exp \left[ -\frac{2 \beta}{\sqrt{1-s}} e^{\frac{z^2}{2(1-s)}} \right] \] \quad \text{Exact solution of the FFP Equation!}
A Formula for the Pinning Probability

\[ P(z, s) = 1 - \exp \left[ -\frac{2\beta}{\sqrt{1-s}} e^{\frac{z^2}{2(1-s)}} \right] \]

Satisfies:

\[
\begin{align*}
\lim_{s \to 1^+} P(z, s) &= 0 \\
\lim_{s \to 1^+} P(0, s) &= 1
\end{align*}
\]

\[ \text{Prob}(z(1) = 0 | z(0) = z_0) = 1 - e^{-2\beta e^{-\frac{z_0^2}{2}}} \]
predicted pinning characteristics

- From the solution (last slide), we see that to first order, the pinning probability should increase linearly in $\beta$ - essentially the $O(1)/\sigma$

- However as $\beta$ increases the pinning probability should saturate

- As $z$ increases the pinning probability should fall off quadratically to lowest order

- The following show unpublished work of my students - actually their PS solutions for this class
PPN graph KO; 1/1/96-1/1/2010; 0.15 pinning criterion

Experimental Finance

KO

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Pinning Probability for All Optionable Stocks

Days from Expiration

2002; all stocks; 0.15
All stocks 2002-2003
All stocks 2002-2003
(log distance with 1 week to expiry in 2nd graph)
Cumulative likelihood of pinning with 1 week to go to expiry (T. MacFarland)
Indices do not pin

Count of Close Price within $0.15 of Strike for 25 AM Settlement Indices

-2 = Thursday for AM Settlement Indices
Market-makers + firm proprietary traders net long

percentage of stocks close within strike price +/- 125, market & Firm net long 43329
**Dimensionless Model for Power-Law Price-Impact Function**

\[
\frac{dS}{S} \propto -\text{const.} \left| \frac{\partial \delta}{\partial t} \right|^p \text{sign} \left( \frac{\partial \delta}{\partial t} \right) dt + \sigma dW
\]

\[
dz = -\frac{\beta \cdot |z|^p \text{sign}(z)}{(1-s)^{3p/2}} e^{-\frac{pz^2}{2(1-s)}} ds + dW
\]

Dimensionless eq. without irrelevant drift terms (alpha=0).
Calculation of Pinning Probabilities by MC Simulation (Gennady Kasyan)

Smooth fit near p=0.5
p=0.5, infinite order phase transition

Pinning under non-linear price-impact models

(i) If \( p \leq 1/2 \), there is no pinning, i.e. \( P[z(1)=0|z(0)=z]=0 \), for all \( z \).

(ii) If \( p > 1/2 \) pinning occurs with finite probability \((<1)\) and

\[
\ln P(z(1)=0 \mid z(0)=z) \propto -\frac{C(\beta, z)}{2p-1}
\]

\[
P_{pin} \propto e^{\frac{C}{2p-1}}, \quad p > 1/2
\]
Real world extensions

• As OI changes with time:
  – Integrate this model

• As other strikes compete:
  – Sum over strikes

• Should work for other instruments that are singly hedged (interest rate, commodity, etc.) but not necessarily indices depending on indirect hedging over multiple instruments
conclusions

• Complex pricing may result from feedback situations

• Here, independent agents (traders) drive the stock price, which in turn alters their hedging behavior, etc., etc.

• Nevertheless simple models work, as long as they are constrained by appropriate boundary conditions

• Allowing the price impact to be a variable leads to the expected result of a phase transition

• Impact functions weaker than square root are suspect—they cannot explain pinning via our mechanism; if they hold for a class of stocks, those stocks will not pin
The Potential Well

Price experiences a force that becomes stronger, more localized near expiration

\[
\frac{dz}{ds} = -\frac{z}{(1-s)^{3/2}} e^{\frac{z^2}{2(1-s)}} \quad (\alpha = 0, \beta = 1)
\]

\(z = \ln(S/K)/(\sigma_\text{ma} \sqrt{\tau})\)
Monte Carlo Simulation of SDE

4000 Paths
Beta ~ 0.1, Alpha=0
Pinning Probability: Dependence on Beta

\[ \beta = \frac{E.OI}{\left( |D| \right) \frac{1}{\sqrt{2\pi\sigma^2T}}} \]

- Increases with OI
- Decreases with volat, expiration
- Decreases with the distance to strike
Pinning Probability: Dependence of Z

\[ z = \ln(S/K)/(\text{vol} \times \sqrt{T}) \]

Beta = 0.1
Effect on Front-Month Option Prices

30-day call prices

Compare B-S with expected value of payoff with respect to new process

30-day implied volatilities

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Pinning

Effect on Second-Month Option Prices

60-day call prices

60-day implied Volatility

Strike ($)
So we have a model which makes strong qualitative and quantitative assertions about the behavior of stocks with large open interests near strikes near expiration.

- There should be a difference between hedgers with long positions and short ones.
- The prices of options on neighboring strikes and series should decline.
- The cumulative probability distribution of pinning should be inversely proportional to the stock volatility.
- The cumulative probability distribution of pinning should be linear in open interest.

- PPN were able to check the difference between long and short using a CBOE database. This information is not available on IVY.
Observations with market-makers net long (∼$0.125)
Market-makers + firm proprietary traders net long
Market-makers net short
Market-makers + firm proprietary traders net short

percentage of stocks close within strike price +/- 125, market & Firms net short 37401

relative trading date from option expiration date
Lecture 3f

Pinning

- First some unfinished business:

- As a market-maker, I know that prices decline around strikes with large open interest. BUT this is anecdotal, not documented. However, if pinning has a non-zero probability of occurring then without price declines, probabilistic arbitrage would exist. Why?

- Binning of stocks by average volatility should be a strong check on our prediction that cumulative probabilities of pinning are inversely related to volatility. This has not been done. But you will do it in the PS!!

- You will check several predictions of our model relating to $\beta$. The PS solutions are actually a strong confirmation of the model’s predictions!

- The drift parameter, $\alpha$, may be subject to an auxiliary condition.

- The basic equation in our model
  \[ \Delta S/S \propto Q \]
  cannot be right in all regimes (large demand/small demand). Would using a more accurate form qualitatively change the solution? It has been suggested that $\Delta S/S \propto Q^\omega$, with $\omega<1$, (a non-analytic form). A qualitative change in the results (at a special value of $\omega$) could be called a phase transition.

- We know now that $\omega=1/2$ is an infinite order phase transition point (unpublished work). NO PINNING below. Thus there are strong arguments against some of Farmer’s assertions!
• Conclusions:

• Pinning of stocks at expiration is a phenomenon well-established statistically in market studies.
• Models which are ad hoc, or involve coordinated behavior by market participants are unsatisfying.
• Equilibrium pricing models (such as BS) are logically and statistically incomplete, ignoring the feedback effect of S/D on underlying prices.
• Delta hedging by independent market-makers is sufficient to account qualitatively for all the observed effects via our model.
• Strong predictions re open interest, volatility and options pricing are made.
• As mentioned before:

  – For $dS/S \sim Q^\omega$, $\omega = 1/2$, there is an infinite-order phase transition. Pinning can only occur for greater than square-root impact functions. This expressly lends doubt to claims by Farmer, et. al.

• The **Impact Function**, $f(Q)$:

  $$f(Q) = dS/S = Q^\omega g(Q),$$

  where $g$ is analytic, has very real practical importance. Suppose you want to sell 1M shares of a stock, XYZ:

  **XYZ** 34.55 34.58 (900x400)
Do you think that you should break the order into many small pieces, or trade it all in one piece?

What you should do depends on whether $\omega$ is greater than or less than 1.

How?