An options primer for the course, Experimental Finance, IEOR E4736
Options Primer

• The subject matter of this course is “event-driven finance”

• An event is a change of trading conditions with a temporal focal point
  • In other words, we have a notion of normal trading conditions, then some event occurs and prices adjust
  • some types of events are earnings announcements, changes in lending rates, corporate actions, etc.
  • It is assumed we have a pricing model which describes the normal trading conditions
  • Then the presence of an event causes prices to change in its vicinity
  • These changes can be both forward in time as well as backwards!
Options Primer

• A pricing model is a blackbox which takes in inputs and outputs the “fair prices” of securities as a function of the inputs
• You are undoubtedly familiar with the most common of these models
• In this course, we will only make reference to Black-Scholes and its discrete cousin: Cox-Ross
• We will think of this model as describing normal trading conditions
• It is important that you review Natenberg if you are insufficiently familiar with equity options or BS
Option Primer

• The inputs for CR (henceforth we will say BS but usually mean CR) are the calendar time, t, the expiration date, T, the discount rate, r, the implied volatility, $\sigma$, the option type, American or European, and the dividend stream. And OF COURSE the stock price, S, and the strike price, K.

• A very important practical fact is that there are multiple interest rates: a long rate, a short rate, a hard-to-borrow rate, broker call, FedFunds, etc...

• When holding an American option would cause future expected returns to fail to exceed the naked stock position the option becomes an exercise. Exercising a call produces + stock, exercising a put – stock
Option Primer

• I have not said long stock and short stock because the exercise contributes to an underlying position in the stock. In other words, if I exercise a call but am currently short 400 shares, my net position becomes short 300 shares.

• You need to demonstrate for yourselves (using put-call parity, described later) that exercising a call is equivalent to selling a synthetic put, while exercising a put is equivalent to selling a synthetic call.

• In margin accounts (where all positions reside with a clearing firm) the value of long securities is charged a long rate; the value of short securities is paid a short rate—unless the security is hard-to-borrow. Cash is paid at whatever rate corresponds to the sign (+/-) of the net value of the position.
Options Primer

• Because the long stock holders *pay* the long rate, a call is generally only an exercise when there is a sufficiently large dividend.

• Puts are generally an exercise when the strike price is high enough.

• In rare occasions the spread between long and short rates or the presence of hard-to-borrowness will lead to calls being an exercise.

• The output of BS are two fair prices: $C(S,K)$ and $P(S,K)$, the call and put prices. Of course, C and S are also functions of all the other inputs mentioned above.
Options Primer

• Since BS takes the same inputs to output both a call price and a put price and because it is demonstrable that $C \equiv P \pmod{F}$, where $F$ is the stock future, we say that there is put-call parity.

• The practical effect of put-call parity is that we may trade puts and calls interchangeably subject to the appropriate hedging.

• While put-call parity strictly holds only for European options, far from the early-exercise boundaries we can assert a functional put-call parity.

• This is because the risk profiles w/o early-exercise are identical for positions which differ only by the replacement of some puts by calls of the same strike and expiry and vice-versa as long as the deltas of the positions are equal.
Options Primer

• The delta, gamma, theta, *vega* of options, also known as the Greeks, are partial differentials of the C and P functions with respect to their various parameter inputs.

• Hence delta is $\frac{\partial C}{\partial S}$, the change in call value as the stock price increases.

• **YOU NEED TO KNOW** delta, gamma, theta, vega VERY WELL – again see Natenberg

• “Inverting” BS means taking the price of an option and inferring the *implied volatility*, $\sigma$, which yields this value (assuming that the additional inputs such as interest rates are understood and agreed to).
Options Primer

• Implied volatilities are the *lingua franca* of finance. When traders and theoreticians speak of volatilities, implied volatilities, vols, etc. they are always stating a value relative to a basic CR model.

• The implied vols for parity options are identical in BS.

• It is therefore useful to enforce a functional put-call parity in CR for American options where we demand that C(S,K) and P(S,K) have identical $\sigma$’s.

• We will use this functional put-call parity to cross-check for bad data as well as to extract hard-to-borrowness.
Options Primer

• Of practical use, a trader will always buy a cheap option to sell an expensive one. If the price of options in parity fluctuates so that the puts become cheaper temporarily or vice-versa there is a money-making opportunity.

• The theoretical meaning of $\sigma$ is this: we imagine a landscape of events which buffet the stock price but whose effect can be viewed as smoothed out in the times of our interest.

• This means that standard option theory is a mesoscopic theory; the time scales of pricing and trading are large wrt these events.
Option Primer

• When we choose to introduce a particular event over a time-scale NOT small in our pricing horizon, then the event produces a non-standard pricing. For example, the announcement of earnings on a specified date will mean that the volatility has a structure involving at least two time scales and the plain, featureless $\sigma$ of BS is insufficient to price options near to earnings.

• Compared to a BS model the prices will diverge. This does not imply tradeability.